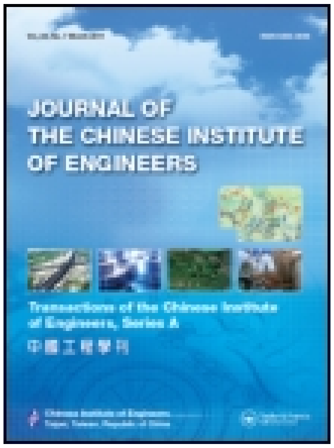

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Fault-distribution-dependent reliable fuzzy control for T-S fuzzy systems with interval time-varying delay

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A new practical actuator-fault model is proposed by assuming that the actuator fault obeys a certain probabilistic distribution. This article addresses the problem of a reliable fuzzy control for Takagi–Sugeno (T-S) fuzzy systems with interval time-varying delays. By using a Lyapunov–Krasovskii approach, a sufficient condition for the existence of a reliable controller is expressed by a set of linear matrix inequalities. Illustrative examples show the effectiveness of the proposed design procedures.

Keywords: reliable control; fault-distribution-dependent; time-varying delay

1. Introduction

Since the pioneering work of Takagi and Sugeno (1985), Takagi–Sugeno (T-S) fuzzy model-based control has been intensively investigated. It combines the flexible fuzzy logic theory and the fruitful linear system theory into a unified framework to approximate complex nonlinear systems, and thus becomes a powerful tool to deal with modeling and control of complex systems, including time-delay systems. Meanwhile, it is well known that time delay often appears in dynamic systems. It is an important source of instability and degradation in control performance. In recent years, the problems of stability and stabilization of the T-S fuzzy systems with time delay have attracted rapidly growing interests (Guan and Chen 2004, Chen *et al*. 2006, Lin *et al*. 2006, Yoneyama 2007, Peng *et al*. 2008, Tian *et al*. 2008, Li *et al*. 2009).

However, all the aforementioned results are yielded by a system which assumes that all control components of the controlled system are in good working condition. In fact, the performance of the closed-loop system might degrade and become unstable when actuator failures occur, as in many practical applications. To improve system reliability and security, a reliable control strategy is usually considered necessary, so that the closed-loop system can operate well, even if faults occur in some control components. Recently, some results have appeared on designing reliable fuzzy control systems (Chen and Liu 2004, Wu and Zhang 2005, 2006, Yang and Cai 2008). Reliable mixed L_2/H_2 fuzzy static output feedback control (Wu and Zhang 2005) and reliable H_∞ control based on fuzzy Lyapunov function and multiple fuzzy Lyapunov functions (Wu and Zhang 2006) have been investigated for use with continuous-time or discrete-time nonlinear systems without time delay. Chen and Liu (2004) present a delay-independent criterion for time-varying delay systems with actuator faults. Reliable control for fuzzy descriptor systems and reliable nonuniform sampling control are studied in Yang and Cai (2008), and the references therein.

Most studies, with regard to reliable control, have depicted the fault model as a scaling factor, such as, in Wu and Zhang (2005), defining $\beta_l \in \Omega \overset{\Delta}{=} \{\beta_l = \text{diag}\times[\beta_{l_1}\beta_{l_2}, \dots, \beta_{l_q}], \quad \beta_{l_i} = 0 \; or \; 1, \quad i = \{1, 2, \dots, q\}, \beta_{l_i} = 0$ corresponds to the case of the *i*-th sensor outage, and $\beta_{l_i} = 1$ denotes no fault in the *i*-th sensor. However, in practical systems, because of actuators aging, zero shift, electromagnetic interference, nonlinear amplification in different frequency fields, etc., the faults vary with circumstance and the components themselves in many cases. It will be more reasonable if the fault scale factor obeys a certain probabilistic distribution in an interval. In fact, there are only two special cases if one lets the factor $\beta_{l_i} = 0 \; or \; 1$. To the best of our knowledge, there are few reliable control results with actuator-fault models which satisfy a certain probabilistic distribution. This has motivated us to investigate problems which more closely resemble real systems.

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The main contribution of this article is that a more practical and general actuator-fault model is established which covers the cases of normal operation, partial degradation, outage and in which the fault obeys a certain probabilistic distribution. Furthermore, a less conservative delay-dependent reliable fuzzy controller is obtained by constructing a new Lyapunov function and using the convexity of the matrix functions in the results derived. The system descriptions and problem formulation are given in Section 2. In Section 3, a linear matrix inequality (LMI)-based method for the design of reliable fuzzy controllers is presented. A numerical example is provided to demonstrate the effectiveness of the proposed method in Section 4. Finally, conclusions are drawn in Section 5.

2. Problem formulation

Consider the T-S fuzzy model with time-varying delay. The *i*-th rule of the model is described by the following IF-THEN form:

R^i: IF $\theta_1(t)$ is $W_1^i, \dots, \theta_n(t)$ is W_n^i,

THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + B_i u(t), \\ x(t) = \phi(t) \quad t \in [-\tau_2, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the input vector; $\phi(t)$ is a continuous vector-valued initial function; $\tau(t)$ denotes the state delay and satisfies $\tau_1 \le \tau(t) \le \tau_2$; W_i^j is the fuzzy set; $\theta_j(t)$ $(j = 1, 2, \dots, n)$ are the premise variables; A_i, A_{di}, and $B_i(i \in \{1, 2, \dots, r\} \triangleq \mathbb{S})$ are constant matrices with compatible dimensions; and $\Delta A_i(t)$ and $\Delta A_{di}(t)$ are unknown matrices of appropriate dimensions satisfying

$$[\Delta A_i(t) \Delta A_{di}(t)] = H_i F_i(t)[E_{1i} E_{2i}], \tag{2}$$

where H_i and $E_{ji}(j = 1, 2, i \in \mathbb{S})$ are known constant matrices of appropriate dimensions and $F_i(t)$ is an unknown matrix function with Lebesgue measurable elements satisfying $F_i^T(t)F_i(t) \le I$.

By using the center average defuzzifier, product inference, and singleton fuzzifier, the global dynamics of T-S fuzzy system (1) can be expressed as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i[(A_i + \Delta A_i(t))x(t) \\ \qquad +(A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + B_i u(t)], \\ x(t) = \phi(t) \quad t \in [-\tau_2, 0], \end{cases} \tag{3}$$

where

$$h_i = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{r} \omega_i(\theta(t))}, \quad \omega_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t)),$$

and $W_j^i(\theta_j(t))$ is the membership value of $\theta_j(t)$ in W_j^i. Therefore, $h_i(\theta(t)) \ge 0$, $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

We consider the following fuzzy state feedback controller for the system (3):

$$u(t) = \sum_{j=1}^{r} h_j K_j x(t), \tag{4}$$

where $K_j \in \mathbb{R}^{m \times n}$ are feedback gain matrices to be determined.

Let $u^F(t)$ represent the control input after faults have occurred. Then, the following fault model is adopted for this study:

$$\begin{aligned} u^F(t) &= \Xi u(t) \\ &= \sum_{i=1}^{m} \sum_{j=1}^{r} h_j \xi_i C_i K_j x(t), \end{aligned} \tag{5}$$

where $\Xi = \mathrm{diag}\{\xi_1, \dots, \xi_m\}$ with $\xi_i(i = 1, \dots, m)$ are m unrelated random variables. It is assumed that ξ_i has mathematical expectation μ_i and variance σ_i^2, respectively, and $C_i = \mathrm{diag}\{\underbrace{0, \dots, 0}_{i-1}, 1, \underbrace{0, \dots, 0}_{m-i}\}$. For convenience, we define $\bar{\Xi} = \mathrm{diag}\{\mu_1, \dots, \mu_m\}$ and $\Delta = \mathrm{diag}\{\sigma_1, \dots, \sigma_m\}$.

Remark 1: There are some papers discussing probabilistic sensor failures for discrete systems. In Wang *et al.* (2005, 2009, 2006) and Hounkpevi and Yaz (2007), the Bernoulli distributed variable γ is used to describe sensor failure, where $\gamma = 0$ and 1 represent the meaning of complete failure or completely normal. In He *et al.* (2009) and Wei *et al.* (2009), the authors assumed that the random variables γ taking values in the interval [0, 1], $0 < \gamma < 1$ means partial failure. However, a fact has been omitted by most researchers when the sensors/actuators have faults, they may result in backward or forward drift, in this case, the sensors/actuators output might be bigger than the real output, which is normal in practical systems; however, it has not drawn much attention till now.

Remark 2: Equation (5) describes a phenomenon of actuator drift by a random matrix Ξ satisfying a certain probabilistic distribution in an interval, ξ_i belongs to the interval $[0, \bar{\xi}]$ with $\bar{\xi} \ge 1$. $\xi_i = 0$ means complete failure of the *i*-th actuator; $\xi_i = 1$ means the *i*-th actuator is in good working condition; $0 < \xi_i < 1$ means partial failure of the *i*-th actuator; and $\xi_i > 1$

means the actuator-amplifier has drifted forward. It should be noted that, in many cases, the gains of actuators could be larger than normal cases because of the surrounding influence or actuator-amplifiers themselves. Therefore, the mathematical expectation μ_i of random variance ξ_i, similar to the scaling factor in Wu and Zhang (2007) should be defined as $0 < \mu_i < \bar{\mu}_i$, where $\bar{\mu}_i \geq 1$. Furthermore, σ_i denotes the gain of actuators fluctuation levels because of influence of all the factors acting on actuators.

Remark 3: $\mu_i = \mathcal{E}\{\xi_i\}$ represents the failure rate of the i-th actuator. It should be noted that with the consideration of the influence of all the factors, $\mu_i = 1$ does not mean the i-th actuator is always in good working condition, the values of ζ_i can be greater or smaller than 1. Simultaneously, $\mu_i = 0$ does not mean the complete failure of the i-th actuator. In particular, if the case $\mu_i = 0$ and $\sigma_i = 0$, simultaneously, it stands for an entire missing of signals, and $\mu_i = 1$, $\sigma_i = 0$ indicates intactness. In fact, actuator-amplifier backward or forward drift usually occurs in practical situations, while complete failure or intactness are two special cases.

By combining Equations (3) and (5), we obtain the following closed-loop system as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [(A_i + \Delta A_i(t))x(t) \\ &\quad + (A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + B_i \Xi K_j x(t)] \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \{(A_i + B_i \bar{\Xi} K_j)x(t) + B_i(\Xi - \bar{\Xi})K_j x(t) \\ &\quad + A_{di}x(t-\tau(t)) + \Delta A_i(t)x(t) + \Delta A_{di}(t)x(t-\tau(t))\},\end{aligned} \tag{6}$$

For convenience, we define $\bar{A}_{ij} = A_i + B_i\bar{\Xi}K_j$ and $\bar{B}_{ij} = B_i(\Xi - \bar{\Xi})K_j$, $\mathcal{G}(t) = \Delta A_i(t) + \Delta A_{di}(t)x(t-\tau(t)$, then Equation (6) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [\bar{A}_{ij}x(t) + \bar{B}_{ij}x(t) + A_{di}x(t-\tau(t)) + \mathcal{G}(t)]. \tag{7}$$

The objective of this study is to develop a reliable fuzzy controller for the closed-loop system considering the stochastic-fault model described by Equation (7). For this purpose, the following lemma derived from Jessen's inequality and definitions are introduced.

Lemma 1 (Gu *et al.* 2003)**:** *For any constant matrix* $R = R^T \in \mathbb{R}^{n\times n}$, $R > 0$, *scalars* $\bar{\tau}$, *and vector function* $\dot{x} : [-r_M, 0] \to \mathbb{R}^n$ *such that the following integration is well defined, it holds that*

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} \dot{x}^T(t+s)R\dot{x}(t+s)\mathrm{d}s \leq \begin{bmatrix} x(t) \\ x(t-\bar{\tau}) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\bar{\tau}) \end{bmatrix}. \tag{8}$$

Lemma 2 (Tian and Peng 2006)**:** *Suppose* M, N, *and* Ω *are constant matrices of appropriate dimensions. Then*

$$(\tau(t) - \tau_1)M + (\tau_2 - \tau(t))N + \Omega < 0 \tag{9}$$

is true for any $\tau(t) \in [\tau_1 \quad \tau_2]$ if and only if

$$(\tau_2 - \tau_1)M + \Omega < 0, \tag{10}$$

$$(\tau_2 - \tau_1)N + \Omega < 0. \tag{11}$$

Definition 1: The system (7) is said to be exponentially stable in the mean-square sense, if there exist constants $\alpha > 0$, $\lambda > 0$, such that $t > 0$,

$$\mathcal{E}\left\{\|x(t)\|^2\right\} \leq \alpha e^{-\lambda t} \sup_{-\tau_2 < s < 0}\left\{\|\phi(s)\|\right\}. \tag{12}$$

Definition 2: For a given function $V : C^b_{F_0}([-\tau_2, 0], R^n) \times S$, its infinitesimal operator $\mathfrak{L}$ (Mao 2002) is defined as

$$\mathfrak{L}V(x_t) = \lim_{\Delta\to 0^+}\frac{1}{\Delta}[\mathcal{E}(V(x_{t+\Delta}|x_t) - V(x_t))]. \tag{13}$$

3. Main result

In this section, we aim to develop an innovative approach to guarantee that the system (7) is exponentially mean-square stable, and the controller K_j can be derived from the following results.

Firstly, we consider the normal case of system (7), i.e., $\Delta A_i(t) = \Delta A_{di}(t) = 0$. In this case, system (7) becomes

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [\bar{A}_{ij}x(t) + \bar{B}_{ij}x(t) + A_{di}x(t-\tau(t))]. \tag{14}$$

Theorem 1: *For given scalars* τ_1, τ_2, μ_i, σ_i *and matrices* K_j $(i, j \in \mathbb{S})$, *the system* (14) *is exponentially mean-square stable if there exist positive-definite matrices*

P, Q_i, $R_i (i=1,2)$, and M_{lij}, $N_{lij} (l=1,2,3,4, i,j \in \mathbb{S})$ such that LMIs (31)–(32) hold.

$$\Pi_{ii}(l) = \begin{bmatrix} \Omega_{ii} & \Pi_i^{21}(l) \\ * & \Pi_i^{22} \end{bmatrix} < 0, \tag{15}$$

$$\Pi_{ij}(l) = \begin{bmatrix} \Omega_{ij} + \Omega_{ji} & \bar{\Pi}_{ij}^{21}(l) \\ * & \bar{\Pi}_{ij}^{22} \end{bmatrix} < 0 \quad (l = 1,2;\ i < j \in \mathbb{S}), \tag{16}$$

where

$$\Omega_{ij} = \begin{bmatrix} \Gamma_{11} & R_1 + N_{1ij} & \Gamma_{13} & -M_{1ij} \\ * & \Gamma_{22} & \Gamma_{23} & -M_{2ij} + N_{4ij}^T \\ * & * & \Gamma_{33} & -M_{3ij} + M_{4ij}^T - N_{4ij}^T \\ * & * & * & -Q_2 - M_{4ij} - M_{4ij}^T \end{bmatrix},$$

$$\Pi_i^{21} = \left[\Psi_{ii}^l\ \Lambda_i^T\ C_{ii}\right],\ \Pi_i^{22} = \operatorname{diag}\{-(\tau_2-\tau_1)R_2, -\mathcal{R}, -\widetilde{\mathcal{R}}\},$$

$$\bar{\Pi}_{ij}^{21} = \left[\Psi_{ij}^l + \Psi_{ji}^l\ \Lambda_{ij}^T\ \Lambda_{ji}^T\ C_{ij}\ C_{ji}\right],$$

$$\Pi_{22} = \operatorname{diag}\{-2(\tau_2-\tau_1)R_2, -\mathcal{R}, -\mathcal{R}, -\widetilde{\mathcal{R}}, -\widetilde{\mathcal{R}}\},$$

$$\Psi_{ij}^1 = (\tau_2-\tau_1)M_{ij},\ \Psi_{ij}^2 = (\tau_2-\tau_1)N_{ij},$$

$$\Gamma_{11} = P\bar{A}_{ij} + \bar{A}_{ij}^T P + Q_1 + Q_2 - R_1,$$

$$\Gamma_{13} = PA_{di} - N_{1ij} + M_{1ij},$$

$$\Gamma_{22} = -R_1 - Q_1 + N_{2ij} + N_{2ij}^T,$$

$$\Gamma_{23} = -N_{2ij} + M_{2ij} + N_{3ij}^T,$$

$$\Gamma_{33} = -N_{3ij} - N_{3ij}^T + M_{3ij} + M_{3ij}^T,$$

$$\Lambda_{ij} = \left[\mathcal{R}\bar{A}_{ij}\ \ 0\ \ \mathcal{R}A_{di}\ \ 0\right],$$

$$C_{ij} = \left[C_{1ij}, \ldots, C_{mij}\right],$$

$$C_{lij} = \left[\sigma_l \mathcal{R} B_i C_l K_j\ \ 0\ \ 0\ \ 0\right]^T,$$

$$\mathcal{R} = \tau_1^2 R_1 + (\tau_2-\tau_1)R_2,$$

$$\widetilde{\mathcal{R}} = \operatorname{diag}\{\underbrace{\mathcal{R}, \ldots, \mathcal{R}}_{m}\}.$$

Proof: Construct a Lyapunov–Krasovskii functional candidate as

$$V(x_t) = \sum_{i=1}^{3} V_i(x_t),$$
$$V_1(x_t) = x^T(t)Px(t),$$
$$V_2(x_t) = \int_{t-\tau_1}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-\tau_2}^{t} x^T(s)Q_2x(s)\mathrm{d}s,$$
$$V_3(x_t) = \tau_1 \int_{-\tau_1}^{0}\int_{t+s}^{t} \dot{x}^T(v)R_1\dot{x}(v)\mathrm{d}v\mathrm{d}s + \int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t} \dot{x}^T(v)R_2\dot{x}(v)\mathrm{d}v\mathrm{d}s.$$

From the definition of Ξ, we can easily see that

$$\mathcal{E}[B_i(\Xi - \bar{\Xi})K_j] = 0. \tag{17}$$

Also, we can have

$$\mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_ih_j\bar{B}_{ij}^T\mathcal{R}\bar{B}_{kl}\right\} \le \mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\bar{B}_{ij}^T\mathcal{R}\bar{B}_{ij}\right\}$$
$$= \mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{m} h_ih_j\sigma_l^2K_j^TC_l^TB_i^T\mathcal{R}B_iC_lK_j\right\}. \tag{18}$$

Using Lemma 1 and the infinitesimal operator (13) for system (14), we have

$$\mathfrak{L}V_1(x_t) = \mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j 2x^T(t)P[\bar{A}_{ij}x(t) + A_{di}x(t-\tau(t))]\right\},$$

$$\mathfrak{L}V_2(x_t) = \mathcal{E}\left\{x^T(t)(Q_1+Q_2)x(t) - \sum_{i=1}^{2} x^T(t-\tau_i)Q_ix(t-\tau_i)\right\},$$

$$\mathfrak{L}V_3(x_t) = \mathcal{E}\left\{\dot{x}^T(t)\mathcal{R}\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s - \int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\right\}$$
$$\le \mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\left\{x^T(t)\left[\bar{A}_{ij}^T\mathcal{R}\bar{A}_{ij} + \sum_{l=1}^{m}\sigma_l^2K_j^TC_l^TB_i^T\mathcal{R}B_iC_lK_j\right]x(t) + x^T(t-\tau(t))A_{di}^T\mathcal{R}A_{di}x(t-\tau(t)) + 2x^T(t)\bar{A}_{ij}^T\mathcal{R}A_{di}x(t-\tau(t)) + \begin{bmatrix} x(t) \\ x(t-\tau_1)\end{bmatrix}^T\begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau_1)\end{bmatrix} - \int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\right\}\right\}.$$

Employing the free-weighting matrix method (Wu *et al.* 2004, Yue and Han 2005), we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j2\zeta^T(t)N_{ij}\left[x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\mathrm{d}s\right] = 0, \tag{19}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j2\zeta^T(t)M_{ij}\left[x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\right] = 0, \tag{20}$$

where

$$\zeta(t) = \left[x^T(t)\quad x^T(t-\tau_1)\quad x^T(t-\tau(t))\quad x^T(t-\tau_2)\right]^T.$$

Note that

$$-2\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\zeta^T(t)N_{ij}\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\mathrm{d}s \le (\tau(t)-\tau_1) \times \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\zeta^T(t)N_{ij}R_2^{-1}N_{ij}^T\zeta(t) + \int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s,$$

$$-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)M_{ij}\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\le(\tau_2-\tau(t))$$
$$\times\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)M_{ij}R_2^{-1}M_{ij}^T\zeta(t)+\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s,$$

where $M_{ij}=[M_{1ij}M_{2ij}M_{3ij}M_{4ij}]$ and $N_{ij}=[N_{1ij}N_{2ij}N_{3ij}N_{4ij}]$.

Hence,

$$\begin{aligned}\mathfrak{L}V(x_t)\le\mathcal{E}\Bigg\{&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)\Big[\Omega_{ij}+\Lambda_{ij}^T\mathcal{R}\Lambda_{ij}+C_{ij}\widetilde{\mathcal{R}}C_{ij}^T\\&+(\tau(t)-\tau_1)N_{ij}R_2^{-1}N_{ij}^T+(\tau_2-\tau(t))M_{ij}R_2^{-1}M_{ij}^T\Big]\zeta(t)\\=\mathcal{E}\Bigg\{&\sum_{i=1}^{r}h_i^2\zeta^T(t)\Big\{\Omega_{ii}+\Lambda_{ii}^T\mathcal{R}\Lambda_{ii}+C_{ii}\widetilde{\mathcal{R}}C_{ii}^T+(\tau(t)-\tau_1)N_{ii}R_2^{-1}N_{ii}^T\\&+(\tau_2-\tau(t))M_{ii}R_2^{-1}M_{ii}^T\Big\}\zeta(t)\\&+\sum_{i,j=1}^{r}\sum_{i<j}h_ih_j\zeta^T(t)\Big\{\Omega_{ij}+\Omega_{ji}+\Lambda_{ij}^T\mathcal{R}\Lambda_{ij}+\Lambda_{ji}^T\mathcal{R}\Lambda_{ji}\\&+C_{ij}\widetilde{\mathcal{R}}C_{ij}^T+C_{ji}\widetilde{\mathcal{R}}C_{ji}^T+(\tau(t)-\tau_1)(N_{ij}+N_{ji})R_2^{-1}(N_{ij}^T+N_{ji}^T)\\&+(\tau_2-\tau(t))(M_{ij}+M_{ji})R_2^{-1}(M_{ij}^T+M_{ji}^T)\Big\}\zeta(t)\Bigg\}.\end{aligned}\tag{21}$$

Using Schur complements and Lemma 2, it can be shown that Equations (15) and (16) are the sufficient conditions for guaranteeing

$$\mathfrak{L}V(x_t)<0.\tag{22}$$

From Equations (15), (16) and (21), the following inequality can be concluded:

$$\mathfrak{L}V(x(t))<-\lambda_{\min}(\Pi_{ij}(l))\mathcal{E}\{\zeta^T(t)\zeta(t)\},\tag{23}$$

where $l=1,\ 2,\quad i,\ j\in\mathbb{S},\quad$ and $\lambda_{\min}$ is the minimum eigenvalue of $\Pi_{ij}(l)$.

Define a new function as

$$W(x_t)=e^{\epsilon t}V(x_t).\tag{24}$$

Its infinitesimal operator $\mathfrak{L}$ is given by

$$\mathcal{W}(x_t)=\epsilon e^{\epsilon t}V(x_t)+e^{\epsilon t}\mathfrak{L}V(x_t).\tag{25}$$

By the generalized Ito formula (Mao 2002), we can obtain from Equation (25) that

$$\mathcal{E}\{W(x_t)\}-\mathcal{E}\{W(x_0)\}=\int_0^t\epsilon e^{\epsilon s}\mathcal{E}\{V(x_s)\}\mathrm{d}s+\int_0^t e^{\epsilon s}\mathcal{E}\{\mathfrak{L}V(x_s)\}\mathrm{d}s.\tag{26}$$

Then, using a method similar to that used in Yue and Han (2005), we can see that there exists a positive number α such that for $t>0$

$$\mathcal{E}\{V(x_t)\}\le\alpha\sup_{-\tau_2\le s\le0}\Big\{\|\phi(s)\|^2\Big\}e^{-\epsilon t}.\tag{27}$$

Since $V(x_t)\ge\{\lambda_{\min}(P)\}x^T(t)x(t)$, it can be shown from Equation (27) that for $t\ge0$

$$\mathcal{E}\{x^T(t)x(t)\}\le\bar{\alpha}^{-\epsilon t}\sup_{-\tau_2\le s\le0}\Big\{\|\phi(s)\|^2\Big\},\tag{28}$$

where $\bar{\alpha}=\alpha/(\lambda_{\min}P)$. Recalling Definition 1, the proof can be completed.

Remark 4: Since the introduction of Lemmas 1 and 2, the convexity of the matrix function is employed to derive the criteria, which can avoid some conservatism caused by enlarging $\tau(t)$ to τ_2 (Yue and Han 2005). For example, in the proof, the time-varying delay $\tau(t)$ appears in $(\tau(t)-\tau_1)N_{ii}R_2^{-1}N_{ii}^T$ and $(\tau_2-\tau(t))M_{ii}R_2^{-1}M_{ii}^T$, by using the convexity property of the matrix functions, Equation (21) is replaced by the equivalent conditions Equations (15) and (16), respectively.

If $B_i=0(i\in\mathbb{S})$, the unforced system (14) can be rewritten as

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j[A_{ij}x(t)+A_{di}x(t-\tau(t))]\\x(t)=\phi(t)\quad t\in[-\tau_2,0]\end{cases}.\tag{29}$$

The following result can be concluded directly from Theorem 1.

Corollary 1: *For given scalars* τ_1,τ_2, *system* (29) *is asymptotically stable if there exist matrices* $P>0,\ Q_i>0,\ R_i>0\ (i=1,\ 2),\ M_i,N_i(i\in\mathbb{S})$ *of appropriate dimensions, such that the following LMIs hold:*

$$\begin{bmatrix}\hat{\Omega}_i+\Phi_i+\Phi_i^T & \hat{\Psi}_i^l & \mathfrak{A}_i^T\mathcal{R}^T\\ * & (\tau_2-\tau_1)R_2 & 0\\ * & * & -\mathcal{R}\end{bmatrix}<0$$
$$(l=1,2;\ i\in\mathbb{S}),\tag{30}$$

where $\hat{\Omega}_i$ is derived by changing the items $\bar{A}_{ij}$, M_{lij} and $N_{lij}(l=1,2,3,4;i,j\in\mathbb{S})$ into A_i, M_i and N_i from Ω_{ij} in Theorem 1, and

$$\hat{\Psi}_i^1=(\tau_2-\tau_1)M_i,\quad\hat{\Psi}_i^2=(\tau_2-\tau_1)N_i,$$
$$\mathfrak{A}_i=\begin{bmatrix}A_i & 0 & A_{di} & 0\end{bmatrix},$$
$$\Phi_i=\begin{bmatrix}0 & N_i & M_i-N_i & -M_i\end{bmatrix}.$$

By a commonly used analytical method for parameter uncertainties, the following result can be obtained for the robust stability of systems (7).

Theorem 2: *For given scalars* $\tau_1,\tau_2,\mu_i,\sigma_i$ *and matrices* $K_j\,(i,j\in\mathbb{S})$, *the system* (7) *is exponentially mean-square stable if there exist positive-definite matrices*

P, Q_i, R_i $(i=1,2)$, and $M_{lij}, N_{lij}(l=1,2,i,j\in\mathbb{S})$ such that LMIs (31)–(32) *hold.*

$$\Pi_{ii}(l)=\begin{bmatrix}\Omega_{ii} & \Pi_i^{21}(l) & \Pi_{ii}^{31}\\ * & \Pi_i^{22} & 0\\ * & * & \Pi_{ii}^{33}\end{bmatrix}<0, \tag{31}$$

$$\Pi_{ij}(l)=\begin{bmatrix}\Omega_{ij}+\Omega_{ji} & \bar{\Pi}_{ij}^{21}(l) & \Pi_{ij}^{31} & \Pi_{ji}^{31}\\ * & \bar{\Pi}_{ij}^{22} & 0 & 0\\ * & * & \Pi_{ij}^{33} & 0\\ * & * & * & \Pi_{ji}^{33}\end{bmatrix}<0$$
$$(l=1,2,i<j\in\mathbb{S}), \tag{32}$$

where

$$\Pi_{ij}^{31}=\begin{bmatrix}H_i^TP & 0 & H_i^T\mathcal{R}^T & 0\\ \varepsilon_{ii}E_{1i} & 0 & \varepsilon_{ii}E_{2i} & 0\end{bmatrix}^T,$$
$$\Pi_{ij}^{33}=\mathrm{diag}\{-\varepsilon_{ii}I-\varepsilon_{ii}I\}.$$

In the following, we propose to design the reliable fuzzy controller gain K_j based on Theorem 1.

Theorem 3: *For given scalars $\tau_i, \iota_i(i=1,2)$, and $\mu_j,\sigma_j\,(j\in\mathbb{S})$, the system* (7) *is exponentially mean-square stable if there exist positive-definite matrices $X, \tilde{Q}_i, \tilde{R}_i(i=1,2)$, and $\tilde{M}_{lij}, \tilde{N}_{lij}, Y_j$ $(l=1,2,i,j\in\mathbb{S})$ such that LMIs* (33) *and* (34) *hold. Furthermore, the reliable fuzzy controller gain $K_j=Y_jX^{-1}$.*

$$\begin{bmatrix}\bar{\Omega}_{ii} & \hat{\Pi}_i^{21}(l) & \hat{\Pi}_{ii}^{31}\\ * & \hat{\Pi}_i^{22} & 0\\ * & * & \hat{\Pi}_{ii}^{33}\end{bmatrix}<0, \tag{33}$$

$$\begin{bmatrix}\bar{\Omega}_{ij}+\bar{\Omega}_{ji} & \hat{\bar{\Pi}}_{ii}21(l) & \hat{\Pi}_{ij}^{31} & \hat{\Pi}_{ji}^{31}\\ * & \hat{\bar{\Pi}}_{ij}^{22} & 0 & 0\\ * & * & \hat{\Pi}_{ij}^{33} & 0\\ * & * & * & \hat{\Pi}_{ji}^{33}\end{bmatrix}<0 \quad (l=1,2,i,j\in\mathbb{S}), \tag{34}$$

where

$$\bar{\Omega}_{ij}=\begin{bmatrix}\bar{\Gamma}_{11} & \bar{R}_1+\bar{N}_{1ij} & \bar{\Gamma}_{13} & -\bar{M}_{1ij}\\ * & \bar{\Gamma}_{22} & \bar{\Gamma}_{23} & -\bar{M}_{2ij}+\bar{N}_{4ij}^T\\ * & * & \bar{\Gamma}_{33} & -\bar{M}_{3ij}+\bar{M}_{4ij}^T-\bar{N}_{4ij}^T\\ * & * & * & -\bar{Q}_2-\bar{M}_{4ij}-\bar{M}_{4ij}^T\end{bmatrix},$$
$$\hat{\Pi}_i^{21}=\left[\bar{\Psi}_{ii}^l\ \bar{\Lambda}_{ii}^T\ \overline{C}_{ii}\right], \hat{\Pi}_i^{22}=\mathrm{diag}\{-(\tau_2-\tau_1)\iota_2,-\varrho X,-\varrho\bar{X}\},$$
$$\hat{\bar{\Pi}}_{ij}^{21}=\left[\bar{\Psi}_{ij}^l+\bar{\Psi}_{ji}^l\ \bar{\Lambda}_{ij}^T\ \bar{\Lambda}_{ji}^T\ \overline{C}_{ij}\ \overline{C}_{ji}\right],$$
$$\hat{\Pi}_{22}=\mathrm{diag}\{-2(\tau_2-\tau_1)\iota_2X,-\varrho X,-\varrho X,-\varrho\bar{X},-\varrho\bar{X}\},$$
$$\hat{\Pi}_{ij}^{31}=\begin{bmatrix}\mu_{ij}H_i^T & 0 & \varrho\mu_{ij}H_i^T & 0\\ E_{1i}X & 0 & E_{2i}X & 0\end{bmatrix}^T,$$
$$\hat{\Pi}_{ij}^{33}=\mathrm{diag}\{-\mu_{ij}I-\mu_{ij}I\},$$
$$\bar{\Gamma}_{11}=A_iX+XA_i^T+B_i\bar{\Xi}Y_j+Y_j^T\bar{\Xi}^TB_i^T+\bar{Q}_1+\bar{Q}_2-t_1X,$$
$$\bar{\Gamma}_{13}=A_{di}X-\bar{N}_{1ij}+\bar{M}_{1ij},$$
$$\bar{\Gamma}_{22}=-t_1X-\bar{Q}_1+\bar{N}_{2ij}+\bar{N}_{2ij}^T,$$
$$\bar{\Gamma}_{23}=-\bar{N}_{2ij}+\bar{M}_{2ij}+\bar{N}_{3ij}^T,$$
$$\bar{\Gamma}_{33}=-\bar{N}_{3ij}-\bar{N}_{3ij}^T+\bar{M}_{3ij}+\bar{M}_{3ij}^T,$$
$$\overline{\Lambda}_{ij}=\left[A_iX+B_i\bar{\Xi}Y_j\ \ 0\ \ A_{di}X\ \ 0\right],$$
$$\overline{C}_{ii}=\left[\overline{C}_{1ij},\ldots,\overline{C}_{mij}\right],$$
$$\overline{C}_{lij}=\left[\sigma_lB_iC_lY_j\ \ 0\ \ 0\ \ 0\right]^T,$$
$$\varrho=\tau_1^2t_1+(\tau_2-\tau_1)t_2\quad \bar{X}=\mathrm{diag}\{\underbrace{X,\ldots,X}_{m}\},$$
$$\bar{\Psi}_{ij}^1=(\tau_2-\tau_1)\bar{M}_{ij},\quad \bar{\Psi}_{ij}^2=(\tau_2-\tau_1)\bar{N}_{ij}.$$

Proof: Defining $X=P^{-1}$, then applying the congruence transformation $\mathrm{diag}\{X,X,X,X,\ \ X,X,\bar{X},I,I\}$ to Equation (31), and setting $\bar{R}_i=XR_iX(i=1,2)$, $\bar{Q}_i=XQ_iX$ $(i=1,2)$, $\bar{M}_{ij}=XM_{lij}X$ $(l=1,2,3,4)$, $\bar{N}_{ij}=XN_{lij}X(l=1,2,3,4)$, $Y_j=K_jX$, and $\mu_{ij}=\varepsilon_{ij}^{-1}$, $R_i=t_iP(i=1,2)$, we obtain that Equation (33) is equivalent to Equation (31).

With a procedure similar to the above, Equation (32) is the sufficient condition for guaranteeing Equation (34).

Remark 5: From Theorem 2, it can be seen that the solvability of LMIs (33) and (34) depend not only on τ_1, τ_2, but also on the distribution of the actuator-fault distribution in a given interval. More information is taken into account in our results compared to the usual fault modeling method mentioned in Section 1.

4. Illustrative examples

In this section, two well-studied examples are used to illustrate the effectiveness of the approaches proposed in this article.

Example 1: Consider the system (29) with the following parameters (Lien *et al.* 2007, Li *et al.* 2009):

$$A_1=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_2=\begin{bmatrix}-1 & 0.5\\ 0 & -1\end{bmatrix},$$
$$A_{d1}=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\quad A_{d2}=\begin{bmatrix}-1 & 0\\ 0.1 & -1\end{bmatrix}.$$

From Table 1, we can see the improvement due to the methods introduced in this article. It can be concluded that the obtained results are less

Table 1. Comparison results for Example 1.

Methods	τ_1	0	0.4
Lien *et al.* (2007)	τ_2	0.831	0.889
Li *et al.* (2009)	τ_2	0.982	1.038
Our results	τ_2	1.265	1.267

conservative than those of Lien *et al.* (2007) and Li *et al.* (2009) in this example.

Example 2: Consider the following time-varying delay system (3) with the parameters (He *et al.* 2009):

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.5-1.5\beta \end{bmatrix},$$

$$A_{d1} = A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \beta = \frac{0.01}{\pi},$$

$$H_1 = H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{11} = E_{12} = \begin{bmatrix} 0 & 0.15 \\ 0 & 0.1 \end{bmatrix},$$

$$E_{21} = E_{22} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix},$$

$$h_1 = \left(1 - \frac{1}{1+\exp(-3(x_2/0.5-\pi/2))}\right)\left(1 - \frac{1}{1+\exp(-3(x_2/0.5+\pi/2))}\right), \quad h_2 = 1-h_1,$$

and $0.01 < \tau(t) < 3$. The following two cases are considered to illustrate the effectiveness of the reliable fuzzy controller when considering the actuator failure occurrence. The standard controller is obtained by assuming the system is normal, and the reliable fuzzy controller is derived while considering actuator failure.

Case 1: We assume the actuators are normal, that is, the parameter Ξ of fault model (5) has expectation $\bar{\Xi} = 1$ and variance $\Delta = 0$, respectively. According to Theorem 2 with $t_1 = 0.26$, $t_2 = 1.2$, we obtain

$$K_1 = \begin{bmatrix} -0.7371 & -0.4181 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.7114 & -0.7893 \end{bmatrix}.$$

Case 2: Assuming that the actuator-fault distribution is given by $\bar{\Xi} = 0.3$, $\Delta = 0.2$, that is, there exist partial actuator failure and fluctuations. According to Theorem 2 with $t_1 = 0.26, t_2 = 1.2$, we get

$$K_1 = \begin{bmatrix} -3.6758 & -2.2136 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -3.7694 & -3.6883 \end{bmatrix}.$$

With the initial conditions $x(t) = [1.8 \quad -0.5]^T$ ($t \in [-2\ 0]$), we assume the stochastic actuator failure described in Case 2 occurs in the interval [4s–35s].

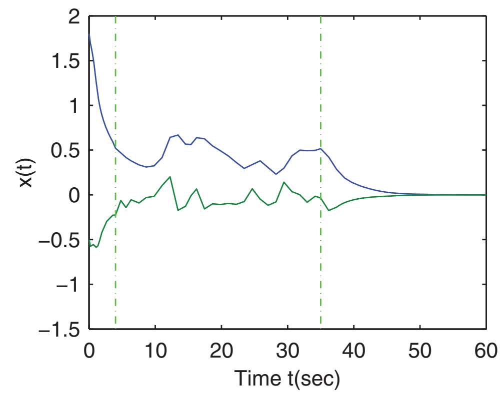

Figure 1. State response under standard controller.

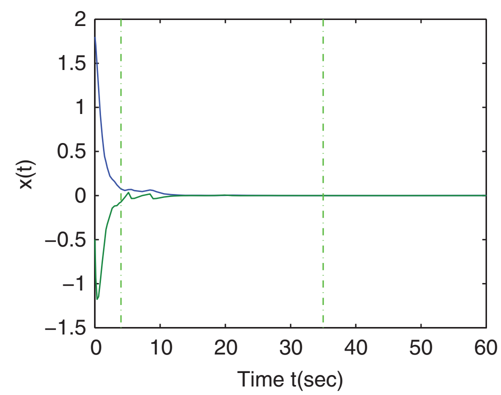

Figure 2. State response under reliable controller.

Figure 1 shows the state response when the system has a standard fuzzy controller, while Figure 2 for a reliable fuzzy controller. It is clear that both the controllers perform very satisfactorily when no failures occur [35s–60s]. Also, it is observed that when an actuator fault occurs, the closed-loop system with the standard controller is not even asymptotically stable, while the closed-loop system using the reliable controller still operates well and maintains an acceptable level of performance.

5. Conclusions

In this article, a new practical actuator-fault model is proposed. We concentrate on the reliable control design problem for a class of T-S fuzzy model based on nonlinear time-varying delay systems, and present a reliable control design methodology to achieve closed-loop stability, not only when the system is operating properly, but also in the presence of certain actuator failures. Numerical examples are given to illustrate the design procedures.

Nomenclature

$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	the set of $n \times m$ real matrices
T	matrix transposition
$X > 0$	the matrix X is real symmetric positive definite
$X \geq 0$	the matrix X is real symmetric positive semi-definite
$\|\cdot\|$	the Euclidean vector norm
$\mathcal{E}\{x\}$	the expectation of stochastic variable x
$\left[\begin{smallmatrix} A & B \\ * & C \end{smallmatrix}\right]$	a symmetric matrix

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